NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 3222

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MEASUREMENT OF HEAT TRANSFER IN THE TURBULENT BOUNDARY LAYER ON A FLAT PLATE IN SUPERSONIC FLOW AND COMPARISON WITH SKIN-FRICTION RESULTS

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MEASUREMENT OF HEAT TRANSFER IN THE TURBULENT BOUNDARY LAYER ON A FLAT PLATE IN SUPERSONIC FLOW AND COMPARISON WITH SKIN-FRICTION RESULTS

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SUMMARY

Local heat-transfer rates on the surface of a heated flat plate at zero incidence to an air stream flowing at Mach numbers of 1.69 and 2.27 are presented. The Reynolds number range for both Mach numbers was 1 million to 10 million. Surface temperatures were maintained near recovery temperature. It was found that the variation of heat transfer with Mach number was in agreement with previously reported variations of directly measured skin friction with Mach number on unheated bodies. The variation with Mach number of the average skin-friction coefficient, as determined from impact-pressure surveys, was in agreement with that from other momentum loss measurements but differed from the variation obtained from directly measured skin friction as reported by others.

INTRODUCTION

It is difficult, on the basis of available basic heat-transfer data pertaining to the turbulent compressible boundary layer, to make valid comparisons of heat transfer and skin friction for equivalent flow conditions. This situation is partially due to the lack of experimental information, and also to the fact that the correlation of the available heat-transfer data on a length Reynolds number basis requires information about the development of the turbulent boundary layer which has not been adequately defined in many of the experiments. Turbulent boundary layers may be induced artificially with trips or may occur naturally after an initial laminar and transitional region; and since the process of development of the boundary layer to a fully turbulent character is not adequately understood, it is necessary to establish that the boundary layer is fully turbulent over the test region and then to fix an effective Reynolds number with which to characterize the actual flow. This effective Reynolds number would then allow correlation of the various heat-transfer tests. It is important to obtain sufficient information to establish an effective Reynolds number when tests are made with small models.

The heat-transfer data of Eber, reference 1, obtained at $M = 2.87$ on a cone-cylinder model for transitional and turbulent flow, vary according to the direction of heat flow. The Nusselt number for the fully turbulent flow varies about ±25 percent from the mean value, and the determination of an effective Reynolds number is impossible. The rocket data presented by Fischer and Norris, reference 2, were obtained in flight and exhibit considerable scatter. The results of Slack, reference 3, Fallis, reference 4, and Monaghan and Cooke, references 5 and 6, are discussed in greater detail in a later portion of this report, but the heat-transfer data are considered briefly here. Slack presents only two values of local heat transfer with accompanying local boundary-layer surveys. An effective Reynolds number cannot be obtained from Fallis' report to enable the correlation of the local heat-transfer results. The average heat-transfer coefficients, as obtained by Monaghan and Cooke, are over an area where the flow is laminar, transitional, and turbulent in character. It may be concluded that these data are insufficient to adequately define the heat transfer over an extended supersonic range.

It is the purpose of this report to present some local heat-transfer data, obtained with a turbulent boundary layer on a heated flat plate at zero incidence to the air stream, for Reynolds numbers of 1 million to 10 million and Mach numbers of 1.69 and 2.27. Temperature recovery factors and average skin-friction coefficients are also presented for these flow conditions. As a final result in this report, comparison is made between the variation of heat transfer with Mach number and the variation of skin friction with Mach number to determine whether or not the ratio of heat transfer to skin friction is essentially unaffected by Mach number as is predicted by Rubesin, reference 7. The data of references 3, 4, 5, and 6 are also used in making this comparison.

NOTATION

C_F average skin-friction coefficient, $\frac{1}{x}\int_0^x c_f dx$

c_f local skin-friction coefficient, $\frac{\text{local shear}}{\frac{1}{2}\rho_1 u_1^2}$

c_p specific heat of air at constant pressure

H boundary-layer form parameter, $\frac{\delta^*}{\theta}$

h local heat-transfer coefficient, $\frac{q}{T_w - T_r}$

k thermal conductivity of air

M	Mach number
P	pressure
Pr	Prandtl number, $\frac{\mu c_p}{k}$
q	local heat-transfer rate per unit area
r	recovery factor, $\frac{T_r - T_1}{T_o - T_1}$
R_x	Reynolds number based on effective distance along plate
R_θ	Reynolds number based on momentum thickness of boundary layer
St	Stanton number, $\frac{h}{\rho_1 u_1 c_p}$
T	absolute temperature
u	air velocity
x	effective distance along plate
x'	actual distance along plate from leading edge
y	distance normal to plate surface
δ	boundary-layer thickness
δ^*	displacement thickness of boundary layer, $\int_0^\delta \left(1 - \frac{\rho u}{\rho_1 u_1}\right) dy$
θ	momentum thickness of boundary layer, $\int_0^\delta \frac{\rho u}{\rho_1 u_1}\left(1 - \frac{u}{u_1}\right) dy$
μ	viscosity of air
ρ	density of air

Subscripts

i	incompressible
av	average value
o	stagnation value
r	condition at surface for zero heat transfer

w condition at surface

1 local free-stream condition at outer edge of the boundary layer

DESCRIPTION OF EQUIPMENT

Wind Tunnel

The Ames 6-inch heat-transfer wind tunnel, which is described in reference 8, was used in this series of tests. The test section had been modified to the half-nozzle configuration as shown in figure 1(a).

Flat-Plate Model

In order to extend the testing region on the flat plate, the model was mounted on the floor of the wind tunnel with the leading edge 7/32 inch above the surface of the straight nozzle block. This particular location was found to be optimum for steady flow at the two test Mach numbers. The air which flowed under the leading edge was bypassed around the test section through ducting back into the wind tunnel at the supersonic diffuser. Various boundary-layer trips were used to induce a turbulent boundary layer over the test area of the plate. A typical trip consisted of a strip of 4/0 garnet paper, 1/2 inch wide by approximately 0.006 inch thick (most of the paper backing had been removed) cemented to the steel plate 1/2 inch back from the leading edge. For each of the lowest Reynolds number runs at the two Mach numbers no trip was used. The spill-over from the turbulent boundary layer, which existed along the lower nozzle block, was sufficient to induce a turbulent boundary layer on the plate surface.

The flat-plate model spanned the test section and was 5-1/2 inches wide and 16 inches long. The plate heaters were made of Advance wire, which was cemented into grooves milled in the top surface of a 1-inch-thick Transite block. The Transite block was cemented to the underside of the steel plate to eliminate any air gaps between the heaters and the steel plate. Twelve heaters were located at 1-inch intervals along the plate. The first heater was 2 inches back from the leading edge. The power input to each heater was controlled by a variable-voltage transformer. A constant-voltage transformer was provided ahead of the heater controls to insure a uniform power source.

Temperature instrumentation consisted of iron-constantan thermocouples soldered in grooves in the steel plate and cemented in the Transite section beneath the plate. A typical cross section A - A, (fig. 1(b)) representative of the 1-inch stations along the plate, shows the various positions at which the temperature could be determined by either direct measurement or by interpolation between measured values.

The location of the thermocouples also provided a measurement of the heat flow sideward in the steel plate and downward through the Transite block. Thermocouples were installed at 1-inch intervals along the center line in the steel and at 2-inch intervals along the center line in the Transite. The thermocouples that were installed 2 inches off the center line in the steel plate and Transite block were positioned at 2-inch and 4-inch intervals, respectively. No air gaps were allowed between the thermocouples and the surrounding material.

Twelve static-pressure orifices, 0.0135 inch in diameter, were located at 1-inch intervals, 1/2 inch off the center line, starting at 2 inches from the plate leading edge.

An impact-pressure survey apparatus was mounted above and downstream of the flat-plate model so that the impact-pressure surveys could be made at the desired test positions. The impact-pressure probe was constructed of flattened hypodermic tubing and had a rectangular opening 0.080 inch by 0.009 inch outside dimensions and 0.075 inch by 0.004 inch inside dimensions. The height of the center line of the probe opening above the plate surface was measured optically.

TEST CONDITIONS AND PROCEDURE

The heat-transfer tests were conducted at average Mach numbers of 1.69 and 2.27 over the range of effective Reynolds numbers from 1 million to 10 million, and at surface temperatures not more than 70° F above recovery temperature. The test conditions are listed in more detail in table I. Two separate measurements of the local heat transfer and of the recovery temperature were made at each condition corresponding to a test number. For the same test conditions as the heat-transfer runs, several impact-pressure surveys of the boundary layer were made in the region of $x' = 3$ to 11 inches. From these surveys were determined the momentum thickness, the effective starting length of the turbulent boundary layer, and the average skin-friction coefficients. This, in brief, is the general procedure followed in this series of tests, but a more detailed account would provide a clearer understanding of the uncertainties in the measurements and results of this experiment and, therefore, is included here.

Initially, for any test run, steady-state conditions of stagnation temperature and pressure were established in the wind tunnel. Stagnation temperature was controllable to ±1/2° F of the set value which was always within 2° F of 118° F. The measured variation during the period in which data were recorded was usually ±1/4° F. The average reading of 20 thermocouples was used to determine the stagnation temperature. The stagnation pressure was maintained to within an average of ±0.29 percent of the desired set value for all runs. Most of this variation was between runs of different days rather than within an individual run.

The free-stream Mach number distribution along the plate was obtained for all the heat-transfer and recovery-temperature runs from the impact and local static pressures. The uncertainty of the free-stream Mach number is estimated to be ±0.005, based on the measurement accuracy of the impact and static pressures. The local Mach number in the free stream was used in the reduction of the data and in the determination of the deviations from true flat-plate flow. Typical Mach number distributions are shown in figure 2. The variation in Mach number was between 1 and 2 percent of the mean Mach number for all runs. It should be noted here that the symbols used to represent the present data in figure 2 and in all subsequent figures, where no notation is given, correspond to those presented in table I.

Heat-Transfer Tests

An effort was made to realize a constant surface temperature along the test region for the heat-transfer runs. Typical plate temperatures are shown in figure 3. This constant temperature ideal was never attained in the region of $x' = 0$ to $x' = 4$ inches, because the nearest heater to the leading edge of the plate was located 2 inches back. The plate temperature was maintained constant to $\pm 0.3^{\circ}$ F for x' positions of from 4 to 12 inches, inclusive. The heat-transfer data that are presented in this report are from the region of from $x' = 4$ to 11 inches, inclusive, because in this region the effects of heat conduction within the model are minimized in the axial x' direction. It was not sufficient to set a constant plate surface temperature initially, without assuring that the model temperatures within the Transite block had reached a steady value. Steady-state temperatures were established within the model before recording the experimental heat-transfer data. An automatic balancing potentiometer with a least count equivalent to 0.067° F was used to measure the emf from the 48 model thermocouples.

The heat input to the heaters was determined from resistance measurements of the Advance wire heaters and from current measurements. These heat inputs were checked with a laboratory type wattmeter. The agreement was within 1.5 percent in the range of power inputs used in these tests. This agreement was considered adequate in view of the fact that the experimental scatter evidenced in the heat-transfer results would completely mask this 1.5-percent power input uncertainty.

Recovery-Temperature Tests

An accurate determination of the recovery temperature is desired when evaluating the local heat-transfer coefficient, especially when the heated-plate-surface temperatures are near to recovery temperature. Two recovery-temperature determinations were made corresponding to each of the test conditions of the heat-transfer runs but with no power to

the heaters. Temperature time histories were used to determine steady-state conditions within the model. All 48 model temperatures were recorded in these runs so that the effects of heat conduction on the surface recovery temperature could be evaluated. The average difference between corresponding surface recovery temperatures from the two runs was 0.4° F and the maximum variation between any two temperatures was 1.4° F. The mean local recovery temperature from the two runs was used in the effective temperature potential for local heat-transfer calculations. It should be noted that the heated-plate temperatures and the recovery temperatures were adjusted to correspond to a common stagnation temperature when determining this temperature potential.

Skin-Friction Tests

For each heat-transfer setting, four or five Mach number surveys were made through the boundary layer at 2-inch intervals along the plate in the region from $x' = 3$ inches to 11 inches. The y distance of the center line of the probe face opening above the plate was determined optically with a cathetometer and a dial gage. Measurements of local static and impact pressures were made for each y position to determine the Mach number. The dial gage used for determining the y position of the probe had a least count of 0.0001 inch. It is estimated that the probe position when off the plate surface could be set to an accuracy of ±0.0005 inch for the low pressure runs and ±0.001 inch for the high pressure runs. Noticeable fluctuations of the impact pressure were evident within the boundary layer, but disappeared at the free stream and solid boundaries. These fluctuations were mechanically damped out of the manometer system when the impact pressure was read.

Mach number surveys of the boundary layer existing on the lower nozzle block were made just ahead of the flat-plate model for the two low Reynolds number runs. The outer 20 percent of the boundary-layer thickness did not bypass beneath the plate leading edge. For all cases, the thickness of the first-measured flat-plate boundary layer was greater than the thickness of the portion of boundary layer which spilled over from the lower nozzle block. No appreciable error should be introduced in the determination of the average skin-friction coefficient from this slight spill-over of nozzle-block boundary layer.

REDUCTION AND EVALUATION OF DATA

Momentum Thickness and Effective Starting Length

The momentum thickness when expressed in terms of free-stream Mach number and static temperature, and local static temperature, T, and Mach number, M, in the boundary layer is

$$\theta = \frac{1}{M_1{}^2}\int_0^{\delta} \left[MM_1 \left(\frac{T_1}{T}\right)^{1/2} - M^2\right] dy$$

The well-known Crocco relations for the boundary layer, which are based on the assumption that Prandtl number is unity, provide an explicit relation for the static temperature in terms of the known quantities T_w, T_1, M, and M_1. It was, therefore, possible to integrate the above expression for the momentum thickness numerically using Simpson's rule. The displacement thickness, δ^*, was obtained in the same way. Since there was a variation in the thickness, roughness, and width of the boundary-layer trips used, it was not possible to correlate the skin-friction and heat-transfer data on the basis of Reynolds number using the actual distance along the plate. The various turbulent-boundary-layer theories (ref. 9) show that the momentum-thickness variation with x' distance is represented by the expression $\theta = Ax'^n$ where n varies between 0.80 and 0.83. The experimental values of $\log \theta$ were plotted against $\log x'$ for each heat-transfer run. The x' distance corresponding to each measured momentum thickness was then shifted a constant amount such that n assumed the value 0.818, which is an average value from the available theories. The amount of shift is designated as the effective starting length, and the new position is called the effective length, x, of the turbulent boundary-layer run. The value of x/x' varies from 0.807 to 1.012 for the $M = 1.69$ tests, and from 0.627 to 0.989 for the $M_1 = 2.27$ tests. The Reynolds number, R_x, based on this effective length and local free-stream properties was used to correlate both the average skin friction and local heat transfer. The average change in R_x for n changing from 0.80 to 0.83 is between 2 and 3 percent.

Average Skin-Friction Coefficient

The average skin-friction coefficient, $C_F = 2\theta/x$, was determined from the calculated θ and effective length x. An estimate of the effect of the free-stream Mach number distribution on the skin-friction coefficient was made using the von Kármán momentum equation

$$C_F = \frac{2\theta}{x} + \frac{2}{x}\int_{x_1}^{x} \frac{(H+2)\ \theta \dfrac{d}{dx}\left(\dfrac{M_1}{\sqrt{1 + 0.2M_1{}^2}}\right)}{\dfrac{M_1}{\sqrt{1 + 0.2M_1{}^2}}} dx + \frac{2}{x}\int_{x_1}^{x} \frac{\theta}{\rho_1}\frac{\rho}{dx} dx$$

The local values of H, θ, and M_1 were known and it was possible to integrate the equation numerically from the point of the initial boundary-layer survey, x_1, to any point in question, x. By utilizing the method of effective starting length to determine the theoretical θ vs. x variation, the flow ahead of the initial boundary-layer survey is

considered as flat-plate flow, regardless of its actual form. The average skin-friction coefficient as determined from the momentum equation, considering measured pressure gradients along the plate, was found to be from 1.2 percent higher to 1.9 percent lower than the flat-plate value for $M_1 = 1.69$ and from 3.1 percent higher to 2.4 percent lower than the flat-plate value for the $M_1 = 2.27$ tests.

Recovery Factor

The temperature recovery factor was calculated from the equation

$$r = \frac{T_r}{T_o}\left(\frac{1 + 0.2\ M_1^2}{0.2\ M_1^2}\right) - \frac{1}{0.2\ M_1^2}$$

using the local values of T_r and M_1 along the center line of the plate. The maximum uncertainty in the recovery factor due only to the estimated uncertainty in measurement of T_r, T_o, and M_1 is 0.0078 and 0.0060 for the Mach numbers 1.69 and 2.27, respectively. The average uncertainty is less than one-half these values.

Heat Transfer

The local heat-transfer coefficient was obtained from the measured heat rate to the heater (corrected for sidewise and downward conduction losses) and the temperature potential based on plate-surface temperature and the measured recovery temperature. The heat flow downward through the Transite block was determined from measurement of two temperatures across the 3/4 inch of known thermal resistance. The downward heat flow was approximately 5 percent of the heater input. The sideward heat flow through the steel plate was accounted for by solving the differential equation describing the heat flow with the appropriate experimental boundary conditions and by assuming that the downward heat flow was constant across the center inch. Since the thermocouples were installed a finite distance from the top of the plate surface, corrections were made to the measured temperatures to give the actual surface temperature. The sideward heat flow and the plate temperature corrections were compensating. The average over-all corrections on the heat-transfer coefficient were a reduction of 2 and 4 percent for $M_1 = 1.69$ and $M_1 = 2.27$ tests, respectively.

When experiments are conducted with relatively small models, it should be established that the boundary layer is fully turbulent along the test region. Two methods were used in this series of tests. One was to compare all the boundary-layer profiles for similarity and type

(i.e., whether laminar, transitional, or turbulent) at each Mach number by plotting M/M_1 vs. y/θ, as is shown for a typical case in figure 4. Another indication of the similarity and type of boundary-layer profiles may be obtained from a comparison of the boundary-layer form parameter $H = \delta^*/\theta$ for all the surveys at each Mach number, as is presented in figure 5. The result from these considerations was to judge all the boundary-layer profiles within the test region to be fully turbulent in nature and similar in form, except one obtained at $x' = 3$ inches. As a consequence, the skin-friction and heat-transfer data in the region associated with this one boundary-layer survey were omitted from the report.

It should be noted that all the properties of air used in the dimensionless groups representing the data in this report are based on the local free-stream conditions existing at the outer edge of the boundary layer.

RESULTS AND DISCUSSION

Inasmuch as the skin-friction and recovery-factor data are basic to a consideration of the heat-transfer characteristics, they will be discussed prior to the main results of this test.

Skin-Friction Results

The average skin-friction coefficients, $C_F/2 = \theta/x$, plotted as a function of R_x are shown in figures 6(a) and 6(b) for $M_1 = 1.69$ and $M_1 = 2.27$ and are compared with the incompressible Kármán-Schoenherr (ref. 10) empirical skin-friction equation. Each set of symbols corresponds to a different heat-transfer-run condition. The figures are presented in this manner to show also the variation in C_F within individual runs. More scatter is evident in the experimental $C_F/2$ for the $M_1 = 1.69$ data than for the $M_1 = 2.27$ tests. Except for two values, the $M_1 = 1.69$ data are within ±6 percent of the mean curve. All of the $M_1 = 2.27$ data are within ±4 percent of the mean curve. The experimental value of the slope of the C_F vs. R_x curves for both Mach numbers is in good agreement with that of the incompressible curve.

Recovery-Factor Results

The calculated recovery factors that are presented as a function of R_x in figures 7(a) and 7(b) exhibit a decrease in value with increasing Reynolds number, although not quite to the extent shown by the flat-plate data of reference 8. The curve from reference 8 represents only the data obtained for tripped boundary layers over a Reynolds number range

from 1 million to 6 million. The mean values of the present data agree to within 0.006 of the results from reference 8, but the x value used in the Reynolds number of reference 8 is the actual distance from the leading edge rather than the effective x distance used here. Use of the effective x value in reference 8 would tend to increase Reynolds number by a small amount and give slightly better agreement.

The commonly used expression for the recovery factor for the turbulent boundary layer, $r = Pr_r^{1/3}$ with physical properties evaluated at the recovery temperature, agrees with experiment for $R_x \approx 6\times10^6$ for both test Mach numbers. With the Prandtl number evaluated at free-stream temperature, $r = Pr_1^{1/3}$ predicts values which are higher than any of the experimental results.

Originally, the arithmetic mean value of the recovery factor at each Mach number was to have been used to calculate the temperature potential in the determination of the local heat-transfer coefficient. However, as noted previously, when the local recovery factor was plotted as a function of R_x, a decrease in r was noted as R_x increased. Maximum discrepancies between runs were also noted in r which would account for recovery temperature differences of 1.4° F and 1.8° F for M = 1.69 and 2.27, respectively. Also, the variation of recovery factor along the plate differed for the lowest and highest Reynolds number runs, which could not be accounted for by considering axial or sidewise heat conduction within the steel plate and Transite block. It was decided, therefore, to use the local recovery temperatures which were actually measured at corresponding test conditions of the individual heat-transfer runs.

Heat-Transfer Results

The basic heat-transfer results of this experiment expressed in terms of local Stanton number as a function of Reynolds number (based on the effective length of the boundary-layer run) are shown in figures 8(a) and 8(b) for the Mach numbers 1.69 and 2.27, respectively.

In figure 8(a), the low-speed data of reference 11 are included as a basis for comparison. Although these low-speed data were obtained on plates having stepwise discontinuous surface temperatures, they were converted to those on an equivalent plate with constant surface temperatures by the theory of reference 12. It is observed that the data are in good agreement with the well-known Colburn equation in the R_x range of 10^5 to 10^6. Since all the supersonic data obtained in this present experiment were within the Reynolds number range of 1 million to 10 million, it was necessary to extrapolate the experimental subsonic results to the higher Reynolds number range to allow a direct comparison.

One method of extrapolation was to use the Colburn equation, which results from the use of the 1/7-power-law skin-friction-coefficient

equation and a modification of the Reynolds analogy, namely $St/(c_f/2) = Pr^{-2/3}$. Another method for comparison is to convert the local skin-friction coefficient, as obtained from the Kármán-Schoenherr equation in the R_x range of 1 million to 10 million, to local Stanton numbers by means of some modification of the Reynolds analogy such as used by Colburn or as is presented in reference 7 by Rubesin. In figure 8(a), it is seen that the agreement between all the alternative methods is fairly good, there being only about a 7-percent difference in the extreme values of the possible choices. It is thus reasonable to assume that the $M_1 = 0$ case is fairly well defined in the R_x range of 1 million to 10 million.

In figure 8(a), it is noted that the data obtained in this experiment at $M_1 = 1.69$ and at temperature potentials of about 70° F are decidedly lower than the extrapolated curves for the $M_1 = 0$ case. A line drawn through the mean of the supersonic data is essentially parallel to the Colburn line and represents the data to a mean error of about 5 percent.[1] This mean line shows a reduction from the $M_1 = 0$ case (Colburn line) of about 22 percent. Similarly in figure 8(b), the mean line through the data obtained at $M_1 = 2.27$ and at temperature potentials of less than 70° F is essentially parallel to the Colburn line and represents the data to a mean error of 5 percent. This mean line for the case of $M_1 = 2.27$ is about 33 percent lower than the extrapolated $M_1 = 0$ case. It was previously noted (see fig. 3) that the plate temperature increased with x' distance over the first 3 or 4 inches. The effect of this positive surface-temperature gradient, $dT_w/dx > 0$, is to increase the heat-transfer coefficient over that of the constant-surface-temperature case. It is difficult to evaluate even the order of magnitude of this effect, say from reference 12, since the character of flow cannot be accurately defined over the region of $x' = 0$ to $x' = 3$ inches because of the location of the boundary-layer trips and the slight spill-over from the lower nozzle block.

The variation of local Stanton number with momentum-thickness Reynolds number R_θ is presented in figures 9(a) and 9(b) for Mach numbers of 1.69 and 2.27, respectively, as an added result of these tests. The correlation is as good as that presented on an R_x basis and is considered more basic because of its dependence on the local characteristics of the boundary layer. Comparison is made with the Van Driest skin-friction theory (ref. 13) combined with the Rubesin analogy (ref. 7). The combined theories give results which are 10.3 and 14.4 percent higher than the measured values for the Mach numbers of 1.69 and 2.27, respectively. This difference is primarily a result of the high skin friction predicted by the Van Driest theory as compared with previous and present experimental values.

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[1]Mean error = $\sqrt{\frac{1}{n}\sum_{n=1}^{n}(St_{meas} - St_{av\ curve})^2}$

Comparison of Skin-Friction and Heat-Transfer Variation with Mach Number

In figure 10, there are shown data taken from reference 14 which indicate the variation of the local or average skin-friction coefficient with Mach number on unheated bodies. The ordinate has been normalized by dividing the skin-friction coefficient by the incompressible value at the corresponding length Reynolds number at which the data were obtained. The curve indicates the variation of skin friction obtained from direct force measurements (filled in symbols) of Coles, Liepmann and Dhawan, and Chapman and Kester. It should be noted that the average skin-friction measurements (open symbols) of Wilson; Rubesin, Maydew, and Varga; Brinich and Diaconis; and the present results made with boundary-layer impact-pressure probes are consistently higher than the force measurements. Further experimental work is required to explain these differences; however, at the present time it is believed that the direct force measurements are the more reliable (see ref. 14) and will be used as a basis for comparison with the heat-transfer results.

In order to properly evaluate all the heat-transfer results as presented in figure 10, a close examination of the test conditions is required. Slack presents only two values of local heat transfer with accompanying boundary-layer surveys obtained on a cooled plate. Since the method of reference 9 was not applicable in this case for determining the effective starting length, the R_x for the corresponding measured R_θ was obtained from the Van Driest theory, and the incompressible Stanton number was obtained for this R_x. The average of the two heat-transfer results, $St/St_i = 0.730$ and 0.759, is plotted in figure 10. (Corresponding test conditions were $T_w/T_1 = 2.00$ and $T_r/T_1 = 2.04$.)

Fallis has measured local heat transfer on a flat plate at near the recovery temperature. Only two of the results can be considered in the turbulent region of flow. These values were obtained at x' positions of 7.75 and 8.75 inches. Preceding these positions was a region of laminar flow up to $x' = 1\text{-}1/2$ inches and a transitional region from $x' = 1\text{-}1/2$ to $6\text{-}3/4$ inches. For the $x' = 8.75$-inch position, there is good agreement of St/St_i with the skin-friction results as shown in figure 10. This agreement is seemingly fortuitous, since the heat-transfer correlation is based on $R_{x'}$ where x' is measured from the leading edge of the plate. The St/St_i result for $x' = 7.75$ inches is 10.4 percent higher. Normally, it is to be expected that the greater the x' value the better agreement with the fully turbulent results, but here, even at the $x' = 8.75$-inch position, 75 percent of the area preceding the test point is either a laminar or transitional region.

The average heat-transfer data of Monaghan and Cooke were obtained at Mach numbers of 2.43 and 2.82 on a heated plate at surface temperatures well above the recovery values. The two results shown in figure 10 are the extrapolated values of $St_{av}/(St_{av})_i$ corresponding to the zero heat-transfer surface temperatures. Both agree well with the directly

measured skin-friction variation. The average Stanton numbers were measured over a heated length of 13.4 inches for both Mach numbers. For the $M_1 = 2.43$ test, the flow was fully turbulent in the region of $x' > 1.8$ inches for the case with maximum heat transfer and $x' > 4.4$ inches for the zero heat-transfer case. For the $M_1 = 2.82$ tests, fully turbulent flow was established in the region of $x' > 4.1$ inches at an intermediate heat-transfer rate and $x' > 7$ inches for the adiabatic case. The position where the fully established turbulent boundary layer begins is not given for the maximum heat-transfer rate, but an estimate of $x' = 2.5$ inches would be approximately correct. Essentially then, the average heat-transfer coefficient is not a true turbulent result but corresponds to flow conditions which are partly laminar, transitional, and turbulent. The excellent agreement of the $St_{av}/(St_{av})_i$ variation with the skin-friction variation would seem to be fortuitous.

Since the surface temperatures used in the present experiment were near the recovery temperature, the variation of Stanton number should depend largely on Mach number alone, thus allowing a direct comparison with the skin-friction data. The two values of St/St_i, shown in figure 10, were defined from the average line drawn through the experimental data of figures 8(a) and 8(b) and the incompressible Colburn equation, and are essentially independent of Reynolds number over the limited R_x range of these tests.

It can be concluded from the agreement of the variation of Stanton number with Mach number, obtained at near recovery temperature, and the variation of directly measured skin-friction coefficient with Mach number on unheated bodies that the ratio of Stanton number to skin-friction coefficient is essentially constant in the range of Mach numbers from 0 to 2.3. This agrees with the predictions of reference 7.

CONCLUDING REMARKS

The local boundary-layer characteristics θ vs. x, and R_θ, may be used directly to correlate the local heat-transfer data (or local skin-friction, if available) or may be used to define an effective Reynolds number, R_x, with which to correlate both local and average heat-transfer and skin-friction results.

The variation with Mach number of the average skin-friction coefficient, C_F/C_{F_i}, as determined from boundary-layer surveys is in agreement with that from other momentum loss measurements but is different from that from directly measured skin friction.

The temperature recovery factor was found to be in good agreement with other flat-plate results over the Reynolds number range of these tests. The recovery factor decreased slightly with increasing Reynolds number, and its mean value may be represented by $r = Pr_r^{1/3}$, with the properties evaluated at recovery temperature.

Based on the local heat-transfer results of this experiment, obtained at near recovery temperature, and from Scesa's ($M \approx 0$) data, the variation of the Stanton number, St/St_i, with Mach number is in agreement with the variation of the directly measured skin friction, C_F/C_{F_i} or c_f/c_{f_i}, on unheated bodies over the Mach number range of $0 < M_1 < 2.3$.

Ames Aeronautical Laboratory
National Advisory Committee for Aeronautics
Moffett Field, Calif., May 6, 1954

REFERENCES

1. Eber, G. R.: Recent Investigation of Temperature Recovery and Heat Transmission on Cones and Cylinders in Axial Flow in the N.O.L. Aeroballistics Wind Tunnel. Jour. Aero. Sci., vol. 19, no. 1, Jan. 1952, pp. 1-6.

2. Fischer, W. W., and Norris, R. H.: Supersonic Convective Heat-Transfer Correlation From Skin-Temperature Measurements on a V-2 Rocket in Flight. Trans. A.S.M.E., vol. 71, no. 5, July 1949, pp. 457-469.

3. Slack, Ellis G.: Experimental Investigation of Heat Transfer Through Laminar and Turbulent Boundary Layers on a Cooled Flat Plate at a Mach Number of 2.4. NACA TN 2686, 1952.

4. Fallis, W. B.: Heat Transfer in the Transitional and Turbulent Boundary Layers of a Flat Plate at Supersonic Speeds. Univ. of Toronto, Institute of Aerophysics, UTIA Rep. no. 19, May 1952.

5. Monaghan, R. J., and Cooke, J. R.: The Measurement of Heat Transfer and Skin Friction at Supersonic Speeds. Part III. Measurements of Overall Heat Transfer and of the Associated Boundary Layers on a Flat Plate at $M_1 = 2.43$. TN Aero. 2129, R.A.E. (British), Dec. 1951.

6. Monaghan, R. J., and Cooke, J. R.: The Measurement of Heat Transfer and Skin Friction at Supersonic Speeds. Part IV. Tests on a Flat Plate at $M = 2.82$. TN Aero. 2171, R.A.E. (British), June 1952.

7. Rubesin, Morris W.: A Modified Reynolds Analogy for the Compressible Turbulent Boundary Layer on a Flat Plate. NACA TN 2917, 1953.

8. Stalder, Jackson R., Rubesin, Morris W., and Tendeland, Thorval: A Determination of Laminar-, Transitional-, and Turbulent-Boundary-Layer Temperature-Recovery Factors on a Flat Plate in Supersonic Flow. NACA TN 2077, 1950.

9. Rubesin, Morris W., Maydew, Randall C., and Varga, Steven A.: An Analytical and Experimental Investigation of the Skin Friction of the Turbulent Boundary Layer on a Flat Plate at Supersonic Speeds. NACA TN 2305, 1951.

10. Schoenherr, Karl E.: Resistance of Flat Surfaces Moving Through a Fluid. Society of Naval Architects and Marine Engineers, Transactions., vol. 40, 1932, pp. 279-313.

11. Scesa, Steve, and Sauer, F. M.: Experimental Investigation of Convective Heat Transfer to Air From a Flat Plate With a Stepwise Discontinuous Surface Temperature. Trans. A.S.M.E., vol. 74, no. 7, Oct. 1952.

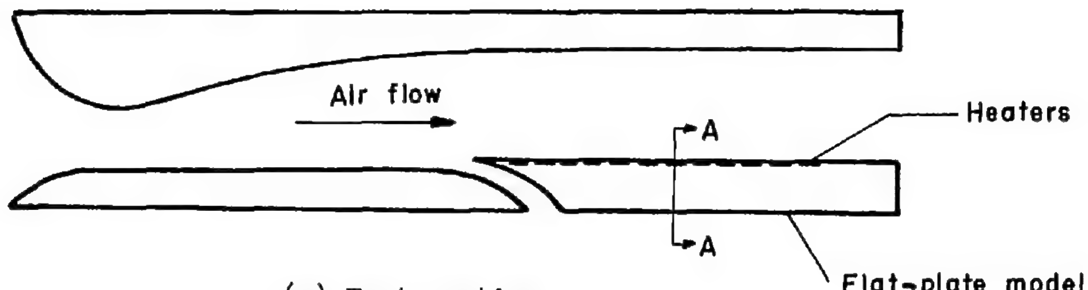
12. Rubesin, Morris W.: The Effect of an Arbitrary Surface-Temperature Variation Along a Flat Plate on the Convective Heat Transfer in an Incompressible Turbulent Boundary Layer. NACA TN 2345, 1951.

13. Van Driest, E. R.: Turbulent Boundary Layer in Compressible Fluids. Jour. Aero. Sci., vol. 18, no. 3, Mar. 1951, pp. 145-160.

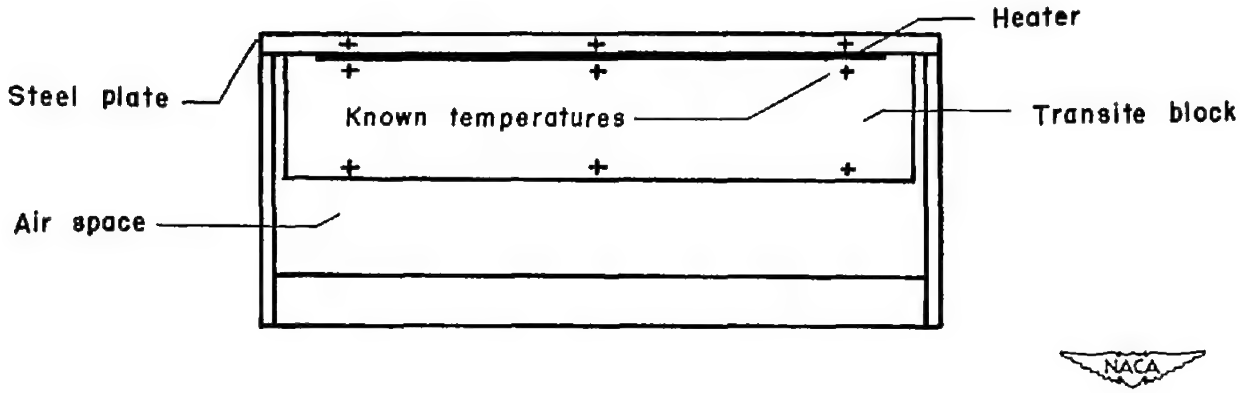
14. Chapman, Dean R., and Kester, Robert H.: Direct-Force Measurements of Turbulent Skin Friction on Cylinders in Axial Flow at Subsonic and Supersonic Velocities. 21st Annual Meeting of Institute of Aeronautical Sciences, Jan. 26-29, 1953. Preprint no. 391, 1953.

TABLE I.- TEST CONDITIONS

Test No.	Symbol	Average P_o, psia	Boundary-layer trip	T_w/T_1	R_x/ft, millions
$M_1 = 1.69$, $T_o = 118^\circ$ F					
1	○	16	None	1.70	4.10
2	□	26	Durite 320 0.50 in. back from x' = 0 0.75 in. wide by 0.01 in. thick	1.65	6.44
3	◇	26	Norton 500 A 0.5 in. back 0.5 in. wide by 0.01 in. thick	1.63	6.42
4	△	42	Norton 500 A 0.5 in. back 0.5 in. wide by 0.01 in. thick	1.61	10.2
$M_1 = 2.27$, $T_o = 118^\circ$ F					
5	◺	30	None	2.18	5.85
6	◺	30	4/0 garnet 0.5 in. back 0.5 in. wide by 0.006 in. thick	2.19	6.14
7	⌓	45	4/0 garnet 0.5 in. back 0.5 in. wide by 0.006 in. thick	2.12	8.58



(a) Test section.



(b) Cross section A-A of flat-plate model.

Figure 1.- Sketch of test section and model.

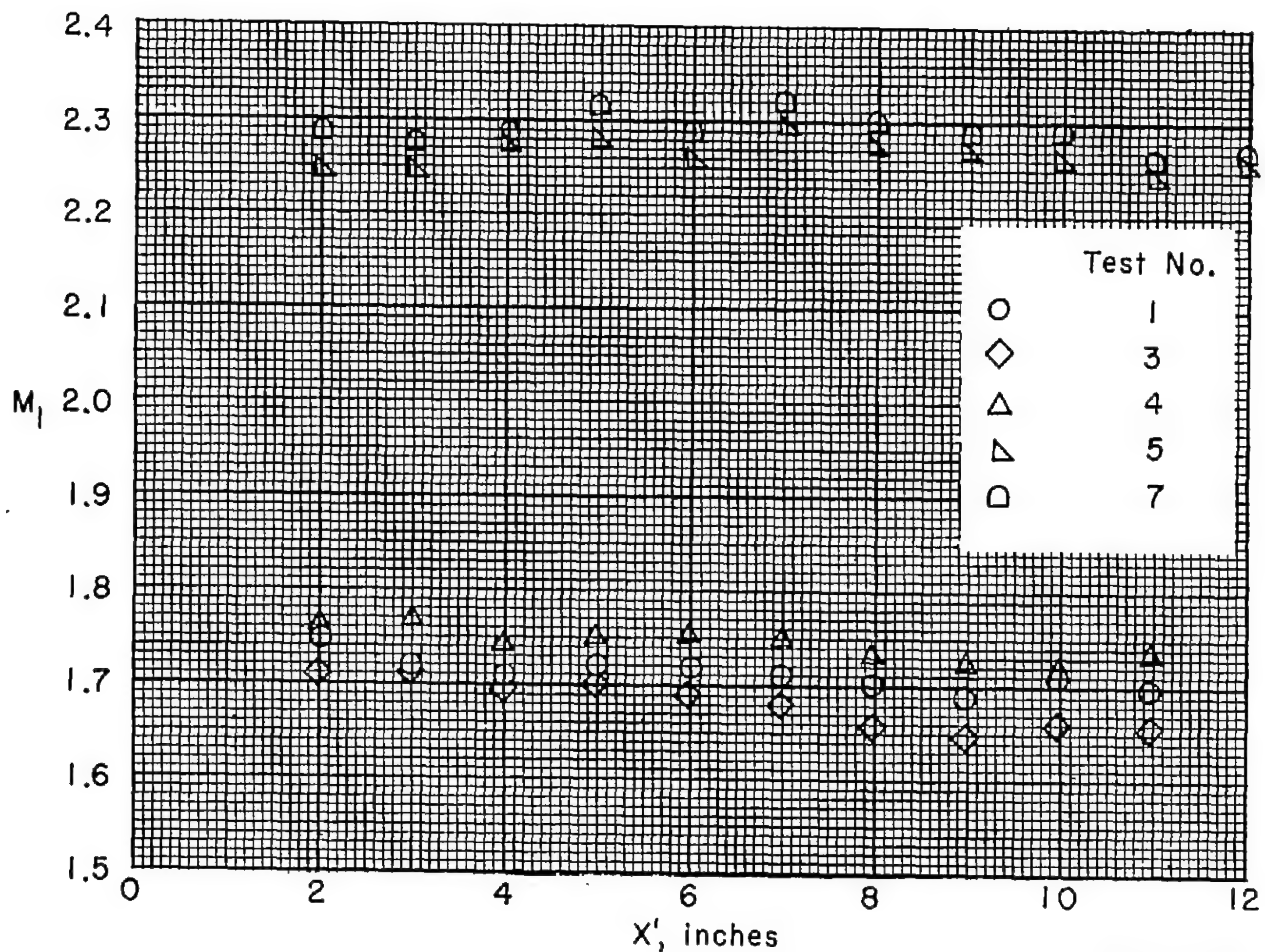


Figure 2.- Typical Mach number distributions along the flat-plate model.

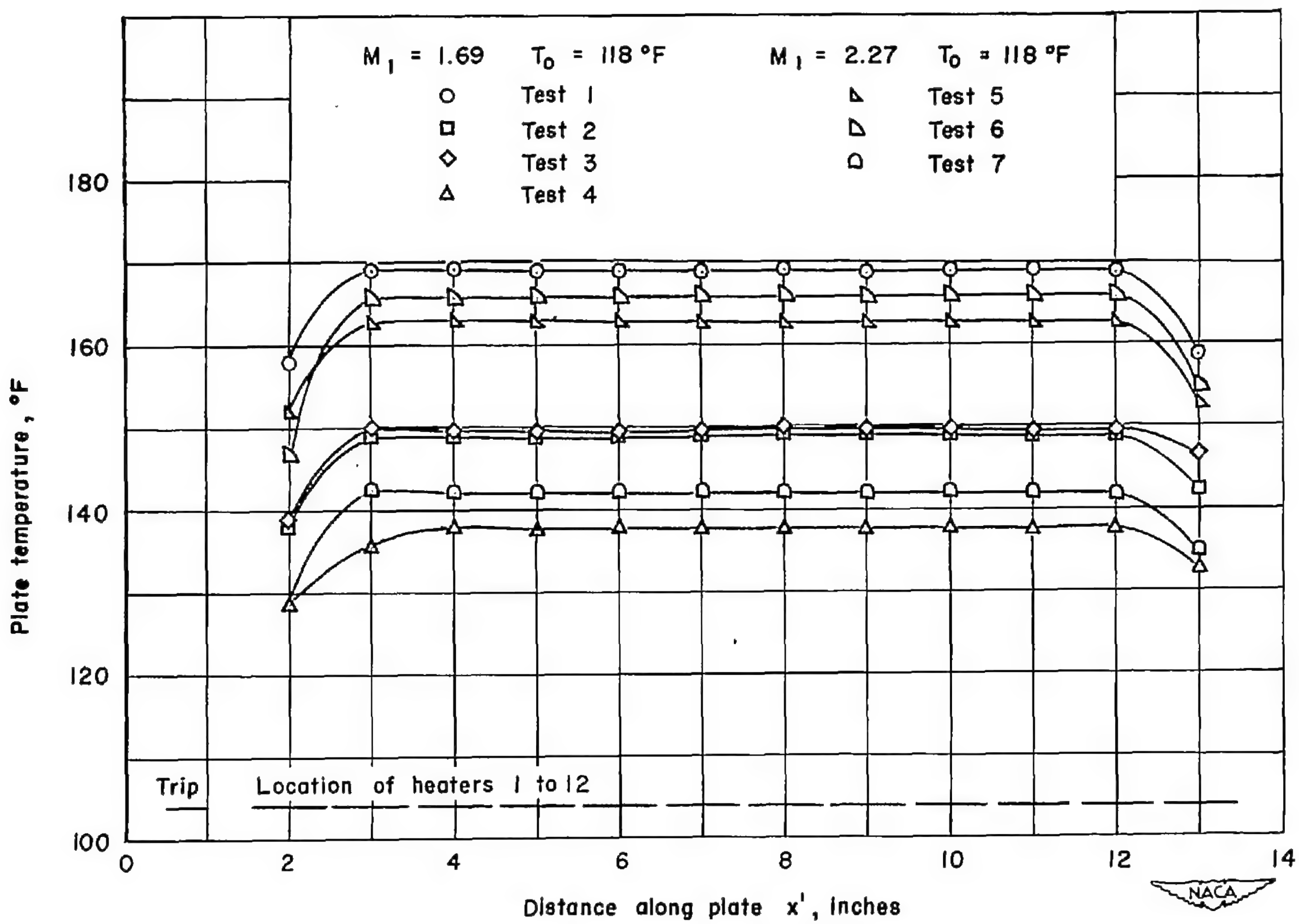


Figure 3.- Typical temperature distributions along flat-plate model.

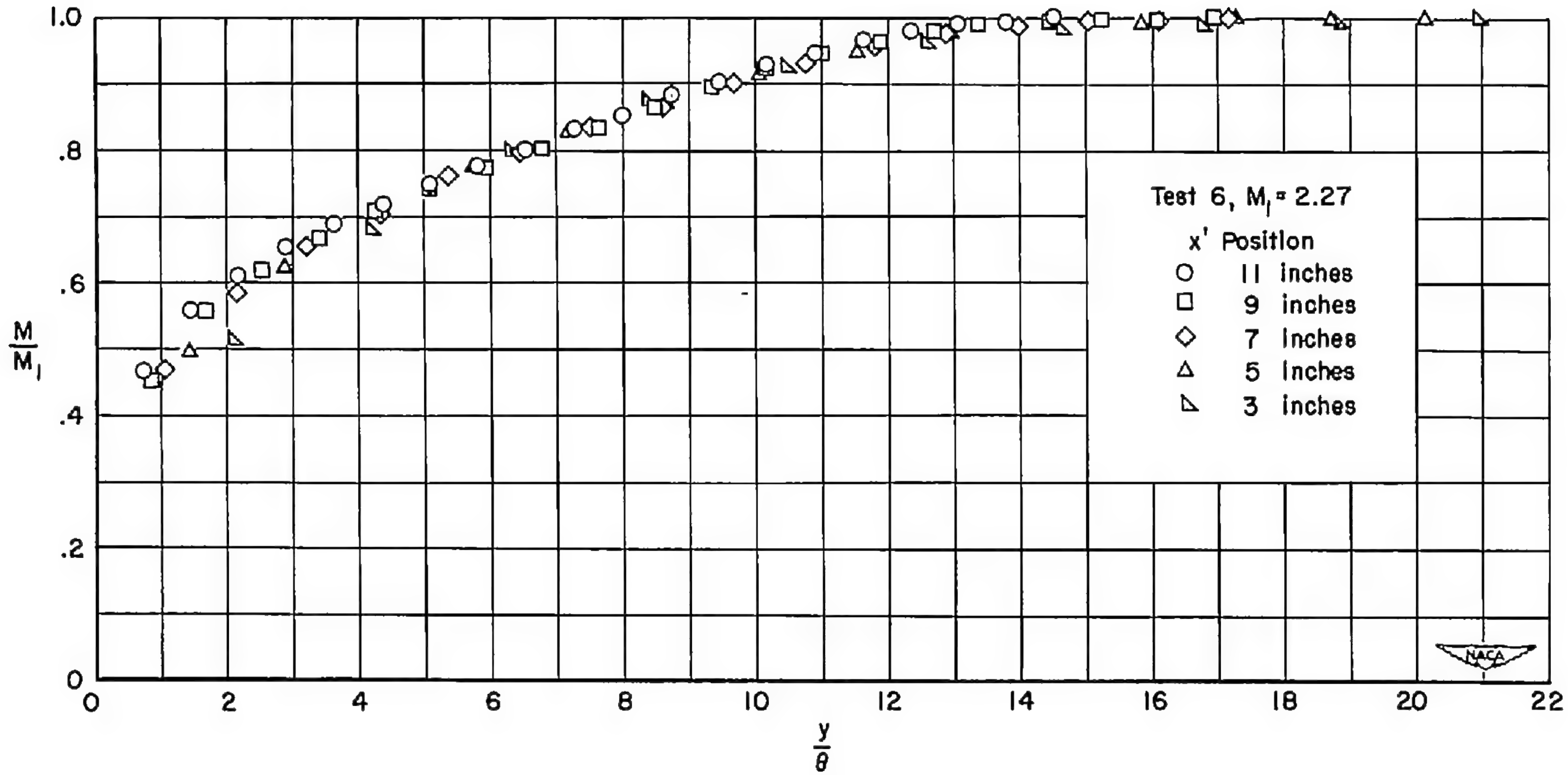


Figure 4.- Typical boundary-layer distributions showing similarity of profiles.

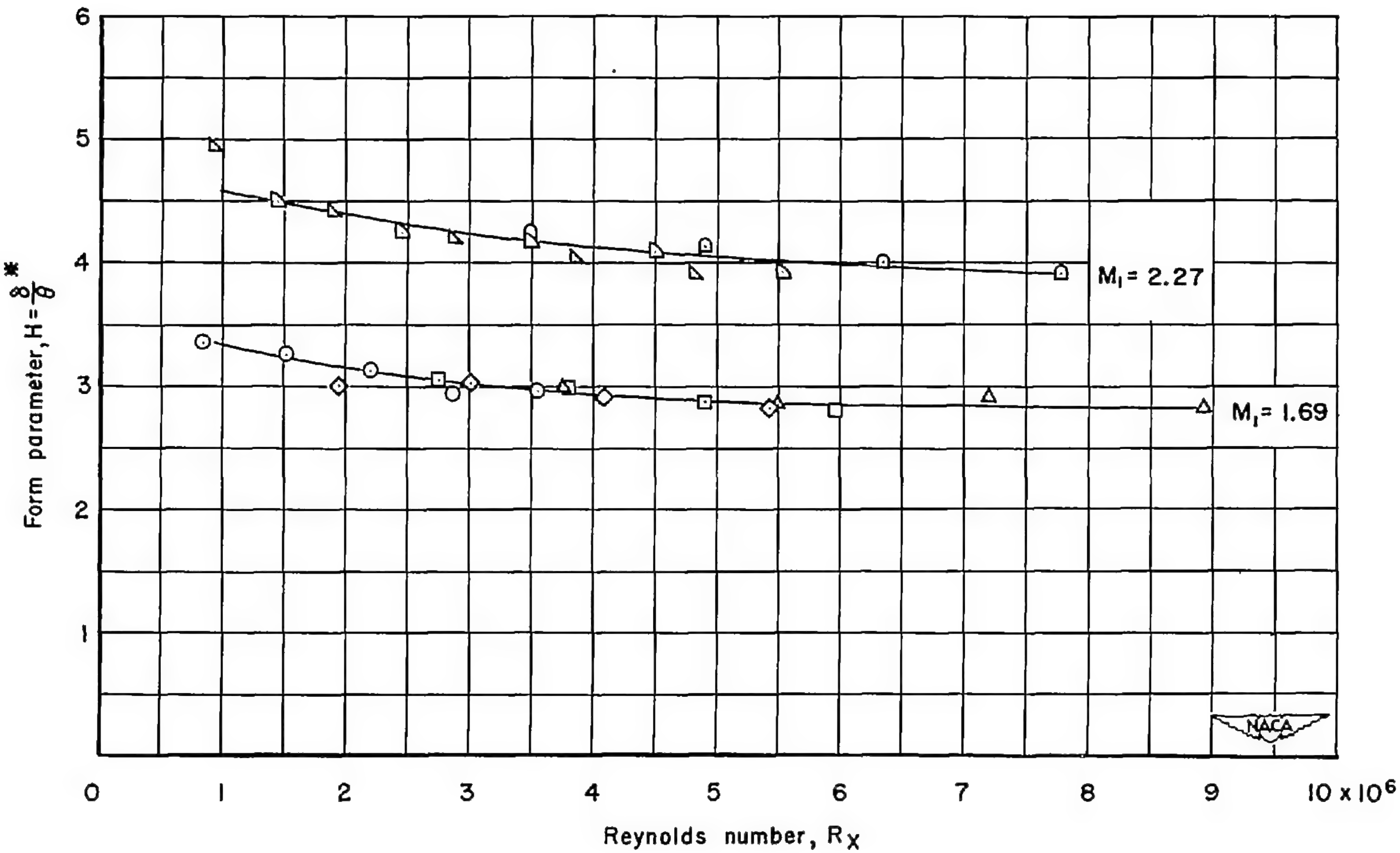
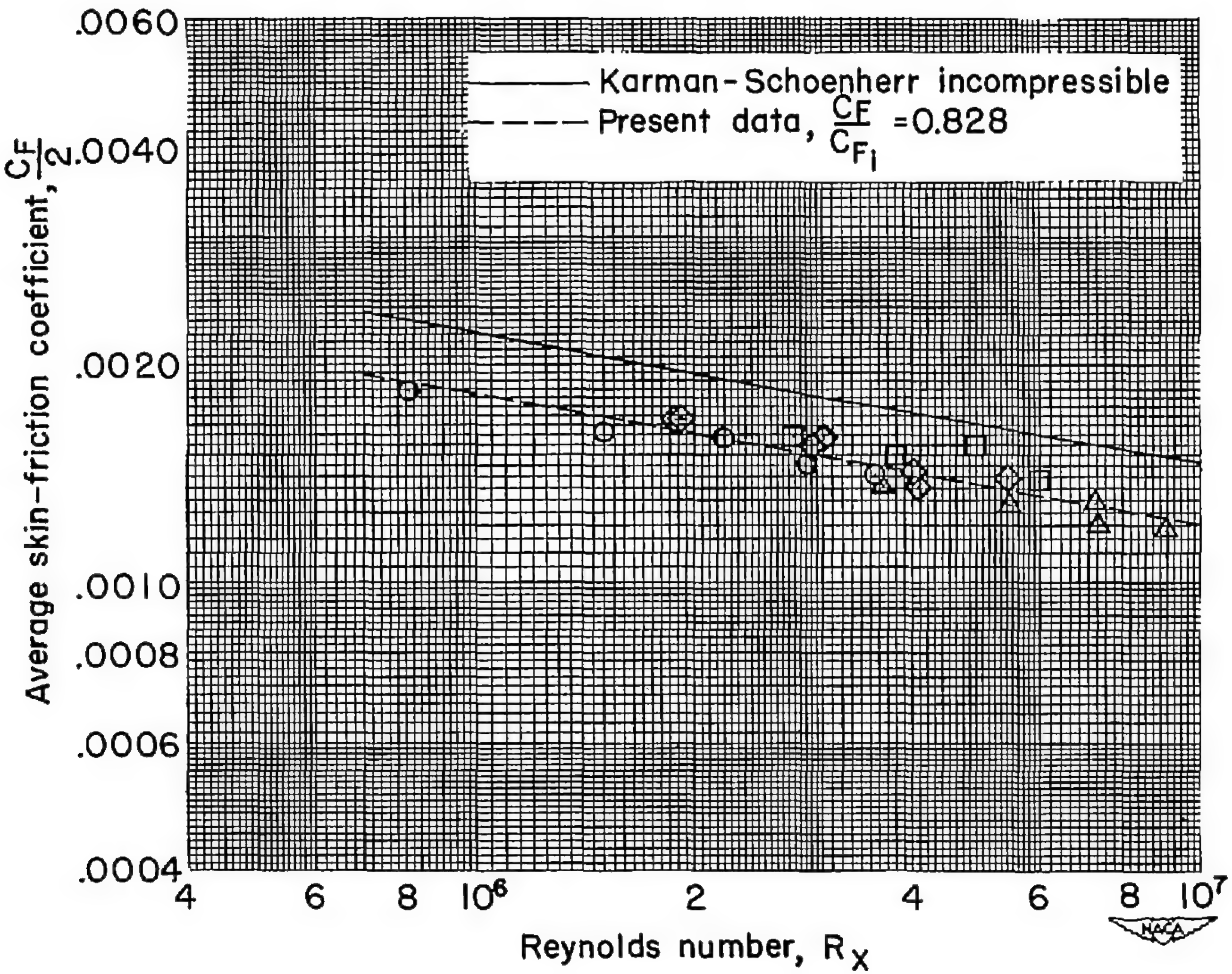
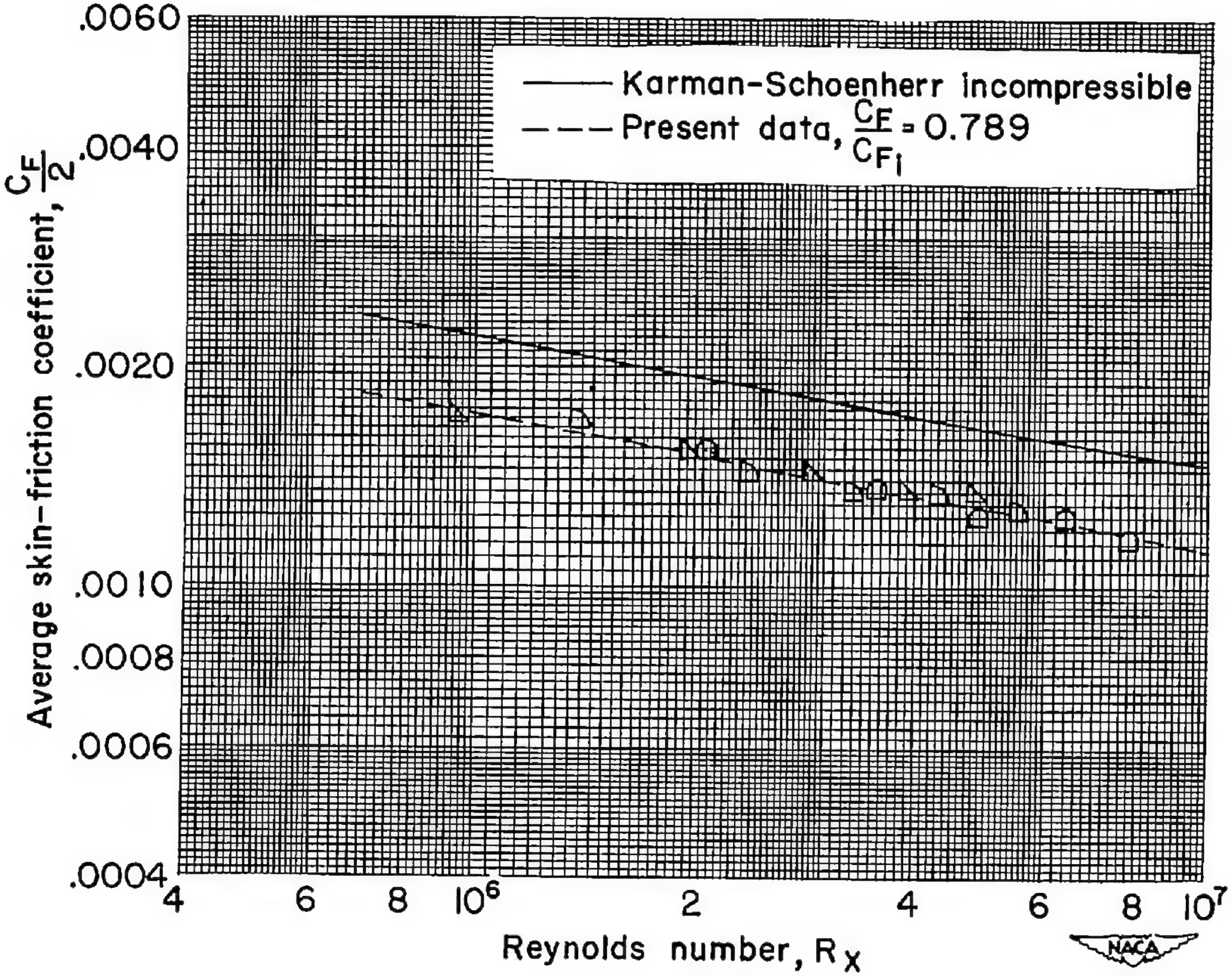


Figure 5.- Variation of boundary-layer form parameter, $H = \delta^*/\theta$, with Reynolds number, R_x.



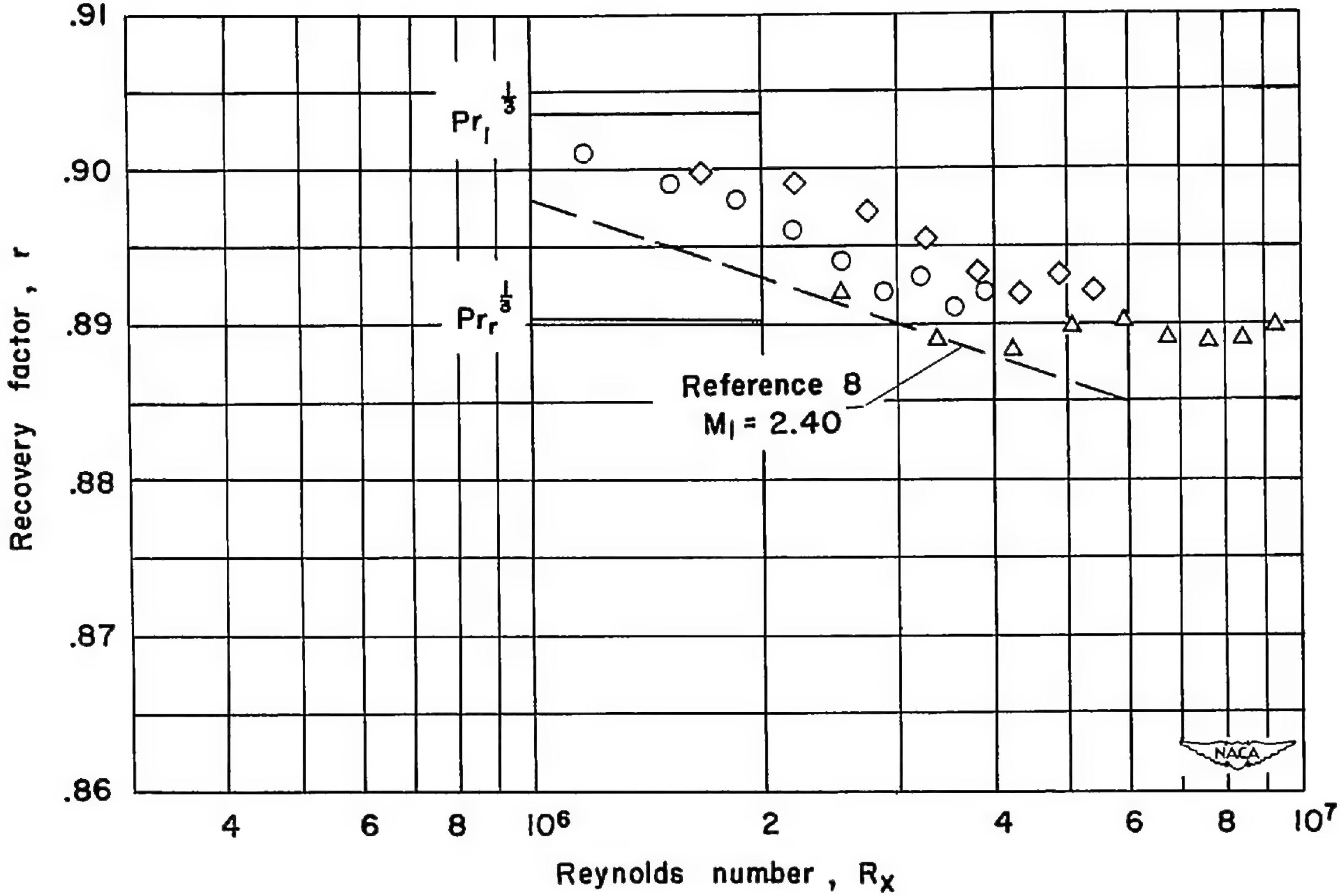
(a) Mach number 1.69.

Figure 6.- Average skin-friction coefficient on flat plate.



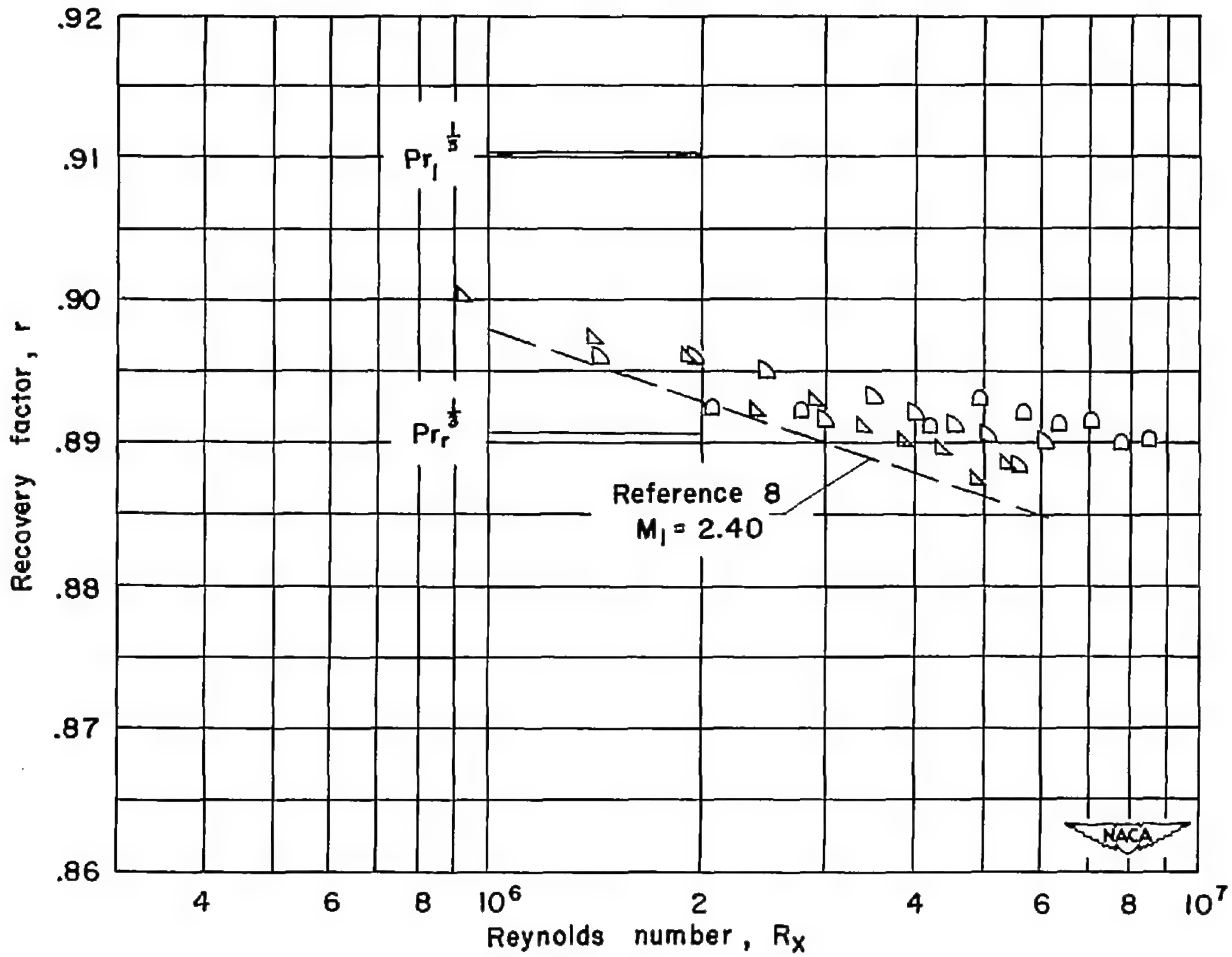
(b) Mach number 2.27.

Figure 6.- Concluded.



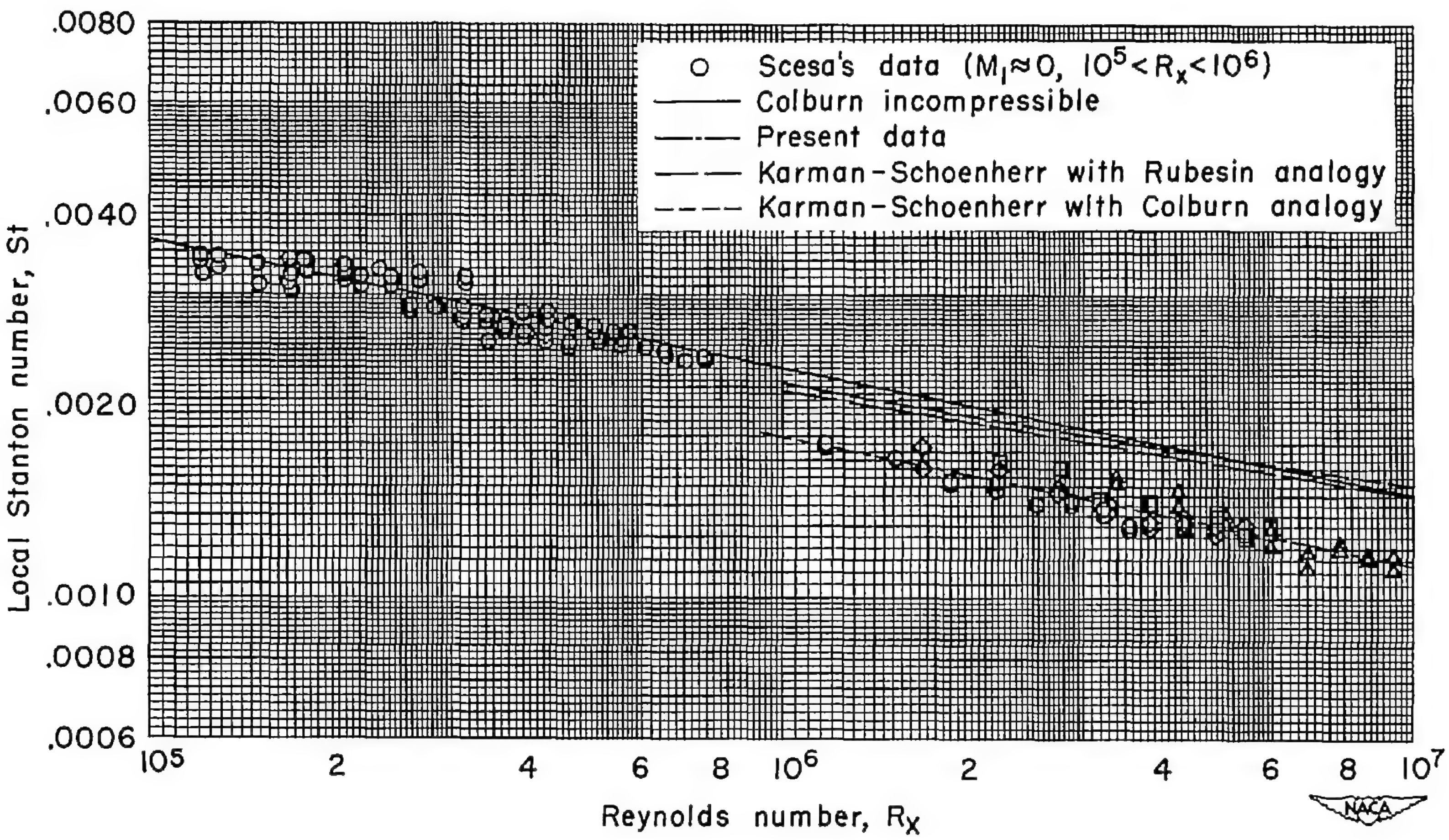
(a) Mach number 1.69.

Figure 7.- Recovery factor in the turbulent boundary layer.



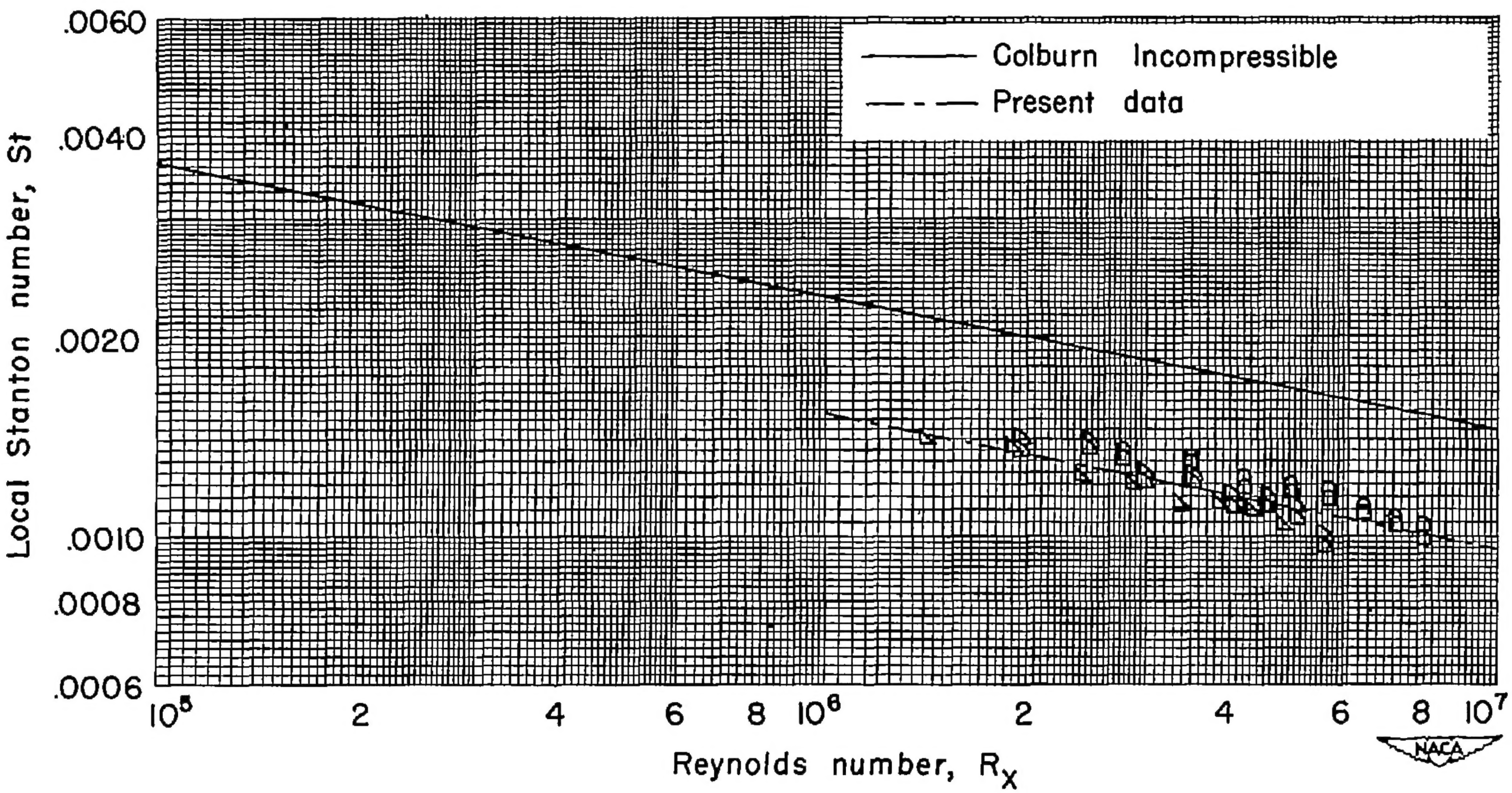
(b) Mach number 2.27.

Figure 7.- Concluded.



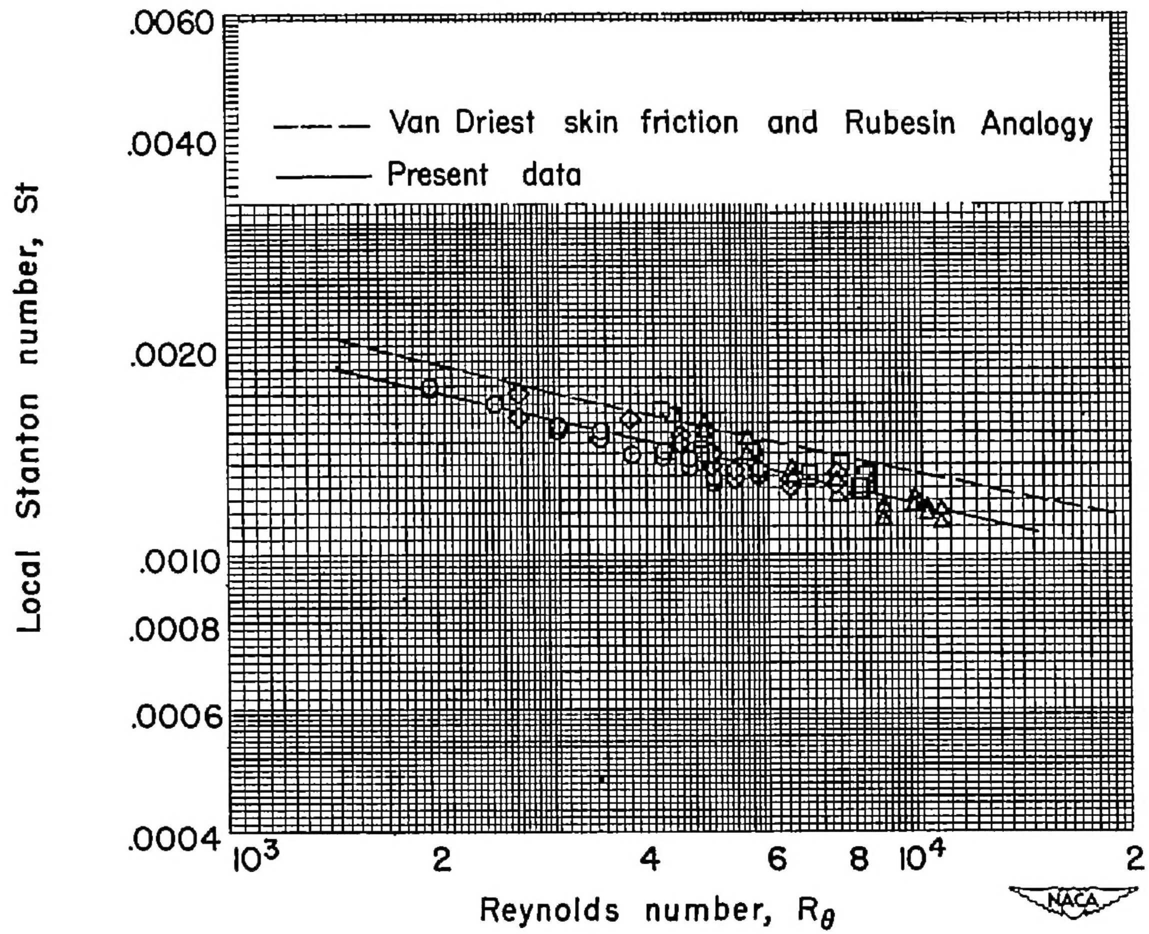
(a) Mach number 1.69, $T_w/T_1 = 1.65$.

Figure 8.- Variation of local Stanton number with Reynolds number.



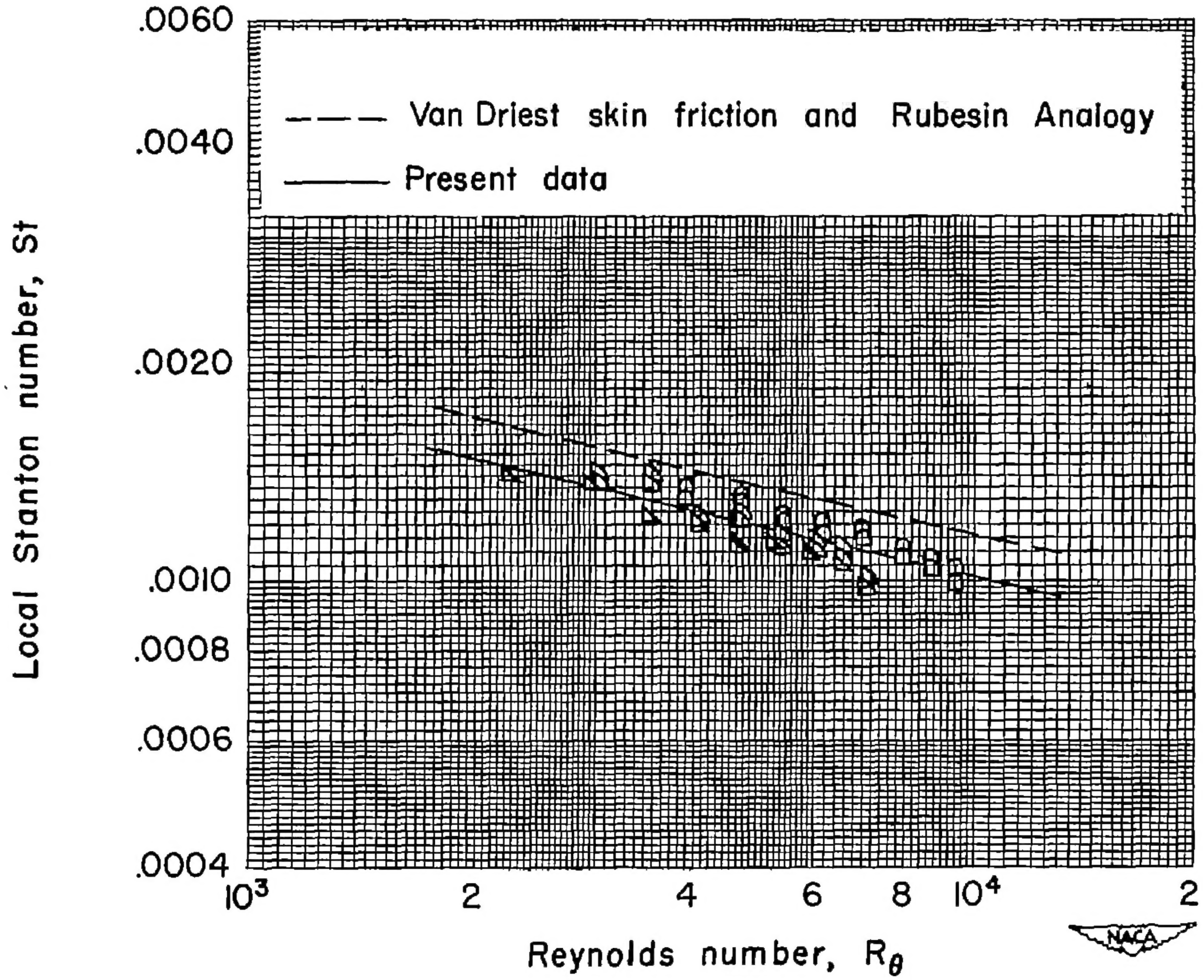
(b) Mach number 2.27, T_w/T_1 = 2.16.

Figure 8.- Concluded.



(a) Mach number 1.69, T_W/T_1 = 1.65.

Figure 9.- Variation of local Stanton number with Reynolds number based on momentum thickness.



(b) Mach number 2.27, T_w/T_1 = 2.16.

Figure 9.- Concluded.

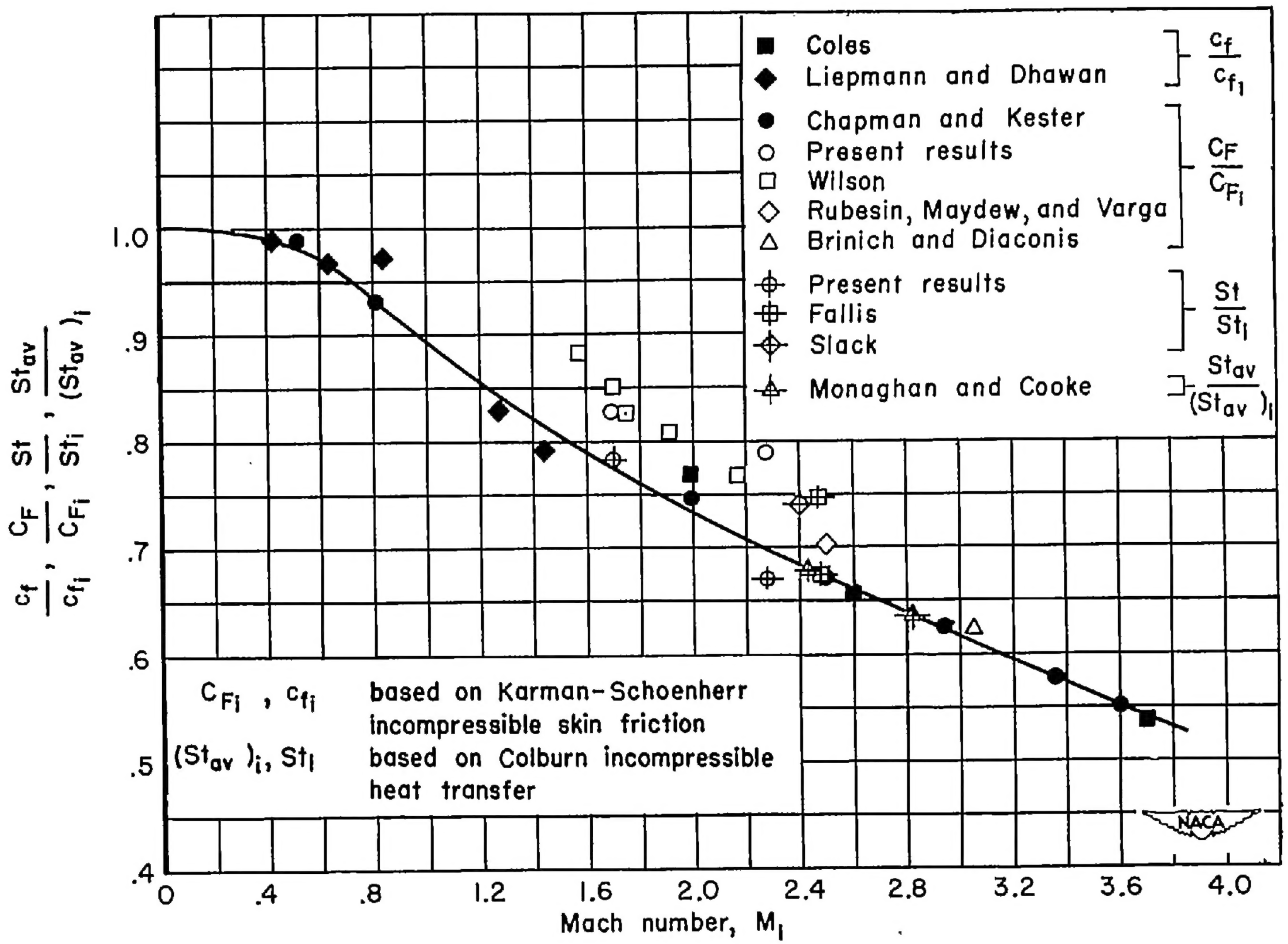


Figure 10.- Comparison of skin-friction and heat-transfer variation with Mach number.